UNITO 2007 EXAM PROBLEMS

(1) If a and b are digits for which

$$\begin{array}{cccc} & & 2 & a \\ \times & & b & 3 \\ \hline & & 6 & 9 \\ & 9 & 2 & \\ \hline & 9 & 8 & 9 \end{array}$$

then $a+b=$

(A) 3 (B) 4 (C) 7 (D) 9 (E) 12

(2) If $6(8x+10\pi)=Q$, then $3(4x+5\pi)=$

(A) $2Q$ (B) $4Q$ (C) $\frac{Q}{2}$ (D) $\frac{Q}{4}$ (E) $\frac{Q}{6}$

(3) Which one of the following points is *not* on the graph of $y=\dfrac{x}{x+1}$?

(A) $(0,0)$ (B) $(-\frac{1}{2},-1)$ (C) $(\frac{1}{2},\frac{1}{3})$ (D) $(-1,1)$ (E) $(-2,2)$

(4) When simplified $\left(-\frac{1}{125}\right)^{-\frac{2}{3}}$ becomes:

(A) $\frac{1}{25}$ (B) $-\frac{1}{25}$ (C) 25 (D) -25 (E) $25\sqrt{-1}$

(5) The arithmetic mean of a set of 50 numbers is 38. If two of the numbers of the set, namely 45 and 55, are discarded, the arithmetic mean of the remaining set of numbers is:

(A) 38.5 (B) 37.5 (C) 37 (D) 36.5 (E) 36

(6) Which of these numbers is largest?

(A) $\sqrt{\sqrt[3]{5\cdot 6}}$ (B) $\sqrt{6\sqrt[3]{5}}$ (C) $\sqrt{5\sqrt[3]{6}}$ (D) $\sqrt[3]{5\sqrt{6}}$ (E) $\sqrt[3]{6\sqrt{5}}$

(7) Find the sum of the arithmetic series

$$20+20\frac{1}{5}+20\frac{2}{5}+\cdots+40$$

(A) 3000 (B) 3030 (C) 3150 (D) 4100 (E) 6000

(8) Which of the following equations have the same graph?

I. $y=x-2$ II. $y=\dfrac{x^2-4}{x+2}$ III. $(x+2)y=x^2-4$

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II and III
(E) None. All the equations have different graphs

(9) Three vertices of parallelogram $PQRS$ are $P(-3,-2)$, $Q(1,-5)$, $R(9,1)$ with P and R diagonally opposite. The sum of the coordinates of vertex S is:

(A) 13 (B) 12 (C) 11 (D) 10 (E) 9

(10) As x varies over all real numbers, the maximum value of $8x - 3x^2$ is:

(A) 0 (B) $\frac{8}{3}$ (C) 4 (D) 5 (E) $\frac{16}{3}$

(11) The ratio of w to x is 3:4, of y to z is 2:3 and of z to x is 6:1. What is the ratio of w to y?

(A) 3 : 1 (B) 3 : 16 (C) 3 : 20 (D) 4 : 27 (E) 1 : 12

(12) In the base ten number system the number 526 means $5 \cdot 10^2 + 2 \cdot 10 + 6$. In the Land of Mathesis, however, numbers are written in base r. Jones purchases an automobile there for 440 monetary units (abbreviated m.u.). He gives the salesman a 1000 m.u. bill and receives, in change, 340 m.u. The base r is:

(A) 2 (B) 5 (C) 7 (D) 8 (E) 12

(13) Let U, N, and O be distinct positive integers such that the product $U \cdot N \cdot O = 2007$. What is the largest possible value of the sum $U + N + O$?

(A) 229 (B) 233 (C) 673 (D) 2009 (E) Such numbers U, N, O don't exist

(14) Medians AD and CE of triangle ABC intersect in M. The midpoint of AE is N. Let the area of triangle MNE be k times the area of triangle ABC. Then k equals:

(A) $\frac{1}{6}$ (B) $\frac{1}{8}$ (C) $\frac{1}{9}$ (D) $\frac{1}{12}$ (E) $\frac{1}{16}$

(15) If the integers from 1 to 2007 are added together, the units' digit of the answer is:

(A) 0 (B) 1 (C) 3 (D) 8 (E) 9

(16) The fourth power of $\sqrt{3 + \sqrt{2 + \sqrt{1}}}$ is

(A) $12 + 6\sqrt{3}$ (B) $3 + \sqrt{3}$ (C) 6 (D) 18 (E) 36

(17) When simplified, the fourth term in the expansion of $\left(\frac{a}{\sqrt{x}} - \frac{\sqrt{x}}{a^2}\right)^6$ is:

(A) $\dfrac{15}{x}$ (B) $-\dfrac{15}{x}$ (C) $-\dfrac{6x^2}{a^9}$ (D) $\dfrac{20}{a^3}$ (E) $-\dfrac{20}{a^3}$

(18) Liquid X does not mix with water. Unless obstructed, it spreads out on the surface of water to form a circular film 0.1 cm thick. A rectangular box measuring 6 cm by 3 cm by 12 cm is filled with liquid X. Its contents are poured onto a large body of water. What will be the radius, in centimeters, of the resulting circular film?

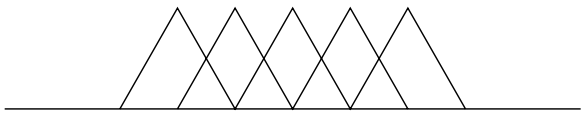
(A) $\dfrac{\sqrt{216}}{\pi}$ (B) $\sqrt{\dfrac{216}{\pi}}$ (C) $\sqrt{\dfrac{2160}{\pi}}$ (D) $\dfrac{216}{\pi}$ (E) $\dfrac{2160}{\pi}$

(19) Let $1, 3, 7, 9, 11, 13, 17, 19, 21, 23, 27, \ldots, 1999, 2001, 2003, 2007$ be the list of all the positive integers, up to 2007, which are odd but don't end in 5. If all these numbers are multiplied out, the units' digit in the final product is:

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

(20) Five equilateral triangles, each with side $2\sqrt{3}$, are arranged so they are all on the same side of a line containing one side of each. Along this line. the midpoint of the base of a triangle is a vertex of the next. The area of the region of the plane that is covered by the union of the five triangular regions is

(A) 10 (B) 12 (C) 15 (D) $10\sqrt{3}$ (E) $12\sqrt{3}$

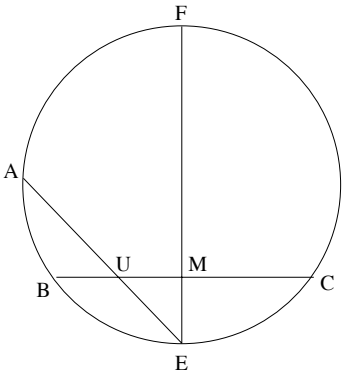


(21) Two tangents are drawn to a circle from an exterior point A; they touch the circle at points B and C, respectively. A third tangent intersects segment AB in P and AC in R, and touches the circle at Q. If $AB = 20$, then the perimeter of triangle APR is:

(A) 42 (B) 40.5 (C) 40 (D) $39\frac{7}{8}$ (E) not determined by the given information

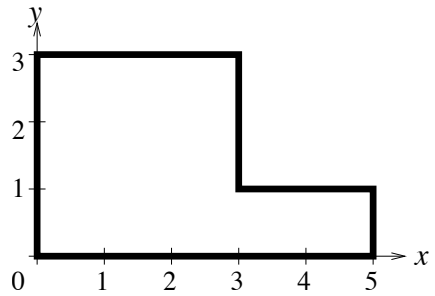
(22) Chord EF is the perpendicular bisector of chord BC, intersecting it in M. Between B and M point U is taken, and EU extended meets the circle in A. Then, for any selection of U, as described, triangle EUM is similar to triangle:

(A) EFA (B) EFC (C) ABM (D) ABU (E) FMC



(23) In the xy-plane, consider the L-shaped region bounded by horizontal and vertical segments with vertices at $(0, 0)$, $(0, 3)$, $(3, 3)$, $(3, 1)$, $(5, 1)$ and $(5, 0)$. The slope of the line through the origin that divides the area of this region exactly in half is

(A) $\frac{2}{7}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{7}{9}$



(24) If the following instructions are carried out by a computer, which value of X will be printed because of instruction 5?

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1. START X AT 3 AND S AT 0.
2. INCREASE THE VALUE OF X BY 2.
3. INCREASE THE VALUE OF S BY THE VALUE OF X.
4. IF S IS AT LEAST 10000,
      THEN GO TO INSTRUCTION 5;
      OTHERWISE, GO TO INSTRUCTION 2 AND PROCEED FROM THERE.
5. PRINT THE VALUE OF X.
6. STOP
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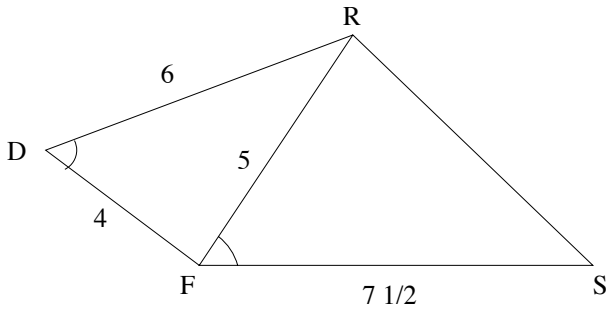
(A) 19 (B) 21 (C) 23 (D) 199 (E) 201

(25) The expression $x^2 - y^2 - z^2 + 2yz + x + y - z$ has:
(A) no linear factor with integer coefficients and integer exponents
(B) the factor $-x + y + z$
(C) the factor $x - y - z + 1$
(D) the factor $x + y - z + 1$
(E) the factor $x - y + z + 1$

(26) In this figure $\angle RFS = \angle FDR$, $FD = 4$ inches, $DR = 6$ inches, $FR = 5$ inches, $FS = 7\frac{1}{2}$ inches. The length of RS, in inches, is:

(A) undetermined (B) 4 (C) $5\frac{1}{2}$ (D) 6 (E) $6\frac{1}{4}$



(27) In the setting of problem 26, all the information stays unchanged except now $FS = 8$ inches. Then the length of RS, in inches, is:

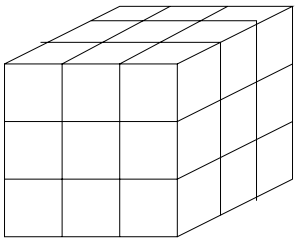
(A) undetermined (B) $6\frac{1}{2}$ (C) $2\sqrt{11}$ (D) $3\sqrt{5}$ (E) 7

(28) Three machines P,Q and R, working together, can do a job in x hours. When working alone P needs an additional 6 hours to do the job; Q, one additional hour; and R, x additional hours. The value of x is:

(A) $\frac{2}{3}$ (B) $\frac{11}{12}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

(29) A large cube is formed by stacking 27 unit cubes. A plane is perpendicular to one of the internal diagonals of the large cube and bisects that diagonal. The number of unit cubes that the plane intersects is

(A) 16 (B) 17 (C) 18 (D) 19 (E) 20



(30) What is the value of the expression

$$\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \cdots + \frac{1}{\log_{100} 100!}?$$

(A) 0.01 (B) 0.1 (C) 1 (D) 2 (E) 10