Department of Mathematics

The University of Texas at San Antonio

Spring 2009 Seminar Series

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Time: 3:00 PM - 4:00 PM

Room: MS 2.02.52

Truncated Moment Problems: The Extremal Case

For a degree $2n$ real d-dimensional multisequence $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{i \in Z_+^d, |i| \leq 2n}$ to have a *representing measure* μ, it is necessary for the associated moment matrix $\mathcal{M}(n)(\beta)$ to be positive semidefinite, and for the algebraic variety associated to β, $\mathcal{V} \equiv \mathcal{V}_\beta$, to satisfy rank $\mathcal{M}(n) \leq$ card $\mathcal{V}$ as well as the following *consistency* condition: if a polynomial $p(x) \equiv \sum_{|i| \leq 2n} a_i x^i$ vanishes on $\mathcal{V}$, then $p(\beta) := \sum_{|i| \leq 2n} a_i \beta_i = 0$. In joint work with Lawrence Fialkow and Michael Möller, we prove that for the *extremal* case (rank $\mathcal{M}(n) =$ card $\mathcal{V}$), positivity of $\mathcal{M}(n)$ and consistency are sufficient for the existence of a (unique, rank $\mathcal{M}(n)$-atomic) representing measure.

The extremal case is inherent in the truncated moment problem; moreover, the existence of a representing measure for $\beta^{(2n)}$ is intimately related to the solution of an extremal truncated moment problem. The new results build on our operator-theoretic approach to truncated moment problems, based on matrix positivity and extension, which, via a "functional calculus" for the columns of the associated moment matrix, allows us to obtain existence theorems in case the columns satisfy one of several natural constraints.

A reception will follow the talk and will be held in MS 2.02.52