Department of Mathematics
The University of Texas at San Antonio

2007 Seminar Series

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Time: 3:00 PM

Room: BSE 2.102

Operator Theoretic Real Algebraic Geometry

Abstract: The subject of real algebraic geometry has flourished in the last few decades notably with elegant algebraic descriptions of when a polynomial **q** on $\mathbf{R}^n$ is positive and on what regions. A theory has developed in the last few years of polynomials whose variables are matrices or operators on a Hilbert space. For example, a noncommutative symmetric polynomial **q** takes positive semidefinite matrix values $\mathbf{q}(X_1, \ldots, X_n)$, on every tuple $(X_1, ..., X_n)$ of matrices if and only if

$$\mathbf{q} = f_1^T f_1 + \cdots + f_n^T f_n.$$

In other words, **q** is a sum of "squares" of polynomials. This is much cleaner than what one sees classically (Hilbert's 17^{th} problem). There are many extensions of the sum of squares results, but the subject takes a turn not found classically. Constraints on curvature force a draconian rigidity: noncommutative "convex" polynomials have degree 2 or less. Beyond this comes the prospect that there is an analogously rigid noncommutative real algebraic geometry. The work originates in trying to develop a theory for organizing the matrix inequalities which are ubiquitous these days in linear engineering systems and control. The talk will give an introduction to some of these topics, hopefully with time to include some of the engineering motivation.

A reception will follow the talk and will be held in BSE 2.102